|  |  |
| --- | --- |
| **Trigonometric Ratios** | **3-Dimensional Cartesian Plane** |
| |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | |  |  |  |  |  |  | |  |  |  |  |  |  | |  |  |  |  |  |  | |  |  |  |  |  |  |   . | A picture containing diagram  Description automatically generated |

**Lecture 5: Vectors**

**Vector Operations**

* The **negative vector of a vector**  is denoted by and it has the same length as but is opposite in direction
* In either case, the magnitude (or length) of is the length of multiplied by the length of . i.e.
* For any vector , the **unit vector** in the direction of is denoted by
* Finally, notice that and are parallel if and only if for some scalar

**Analytic Representation**

* Consider the Cartesian plane with origin . For any , we can write its **position vector** as
* Furthermore, consider , for any, we can write its position vector as
* For and we find:

, and

**Standard Unit Basis Vectors**

* and in 2 space and and and in 3 space
* For any ,

**Dot Product**

* The **dot product** (or **scalar product)** of and is
* Consider and with an angle between them, . Suppose we locate both vectors with their initial points at the origin, and if and , then
* We say that and are **orthogonal** (or perpendicular) if the angle between them is .
* In this case,

**Projection:** The **scalar projection** of on is andthe **vector projection** of on is

**Work Done by a Force:**

**Direction Cosines**

* Consider a vector and let , and be the angles which makes with , and , respectively
* Then , and are the **direction cosines** of
* We have , , Therefore,

**Lecture 6: Vectors & Introduction to Matrices**

**Cross Product**

* The **cross product** of and is
* is called the **scalar triple product of** , and
* is called the **vector triple product of** , and
* , and are **coplanar** (lie in the same plane) if and only if

|  |  |  |
| --- | --- | --- |
| **Area of a Parallelogram** | **Area of a Triangle** | **Volume of a Parallelepiped** |
|  |  |  |

|  |  |  |
| --- | --- | --- |
| **Matrix Addition** | **Scalar Multiplication** | **Matrix Multiplication** |
|  |  | where |

* Valid properties of matrix multiplication: (1) (2) (3)
* The **transpose** of an matrix is the matrix where
* We say that a square matrix is **symmetric** if
* Note: (1) (2) (3)

**Identities and Inverses**

* An **identity matrix** is a square matrix with all main diagonal entries equal to and all other entries equal to
* E.g. , ,
* Note: is the **identity element** of matrix multiplication, i.e. for , and
* If is square and of order , and if there exists a such that , then we say that **is invertible** and we call the **inverse** of and we write to denote the inverse

**Lecture 7: Systems of Linear Equations & Gaussian Elimination**

|  |  |  |
| --- | --- | --- |
| **Linear system of** **equations in** **variables** | **Matrix of system** | **Augmented matrix of system** |
| … | i.e. | i.e. |

* A system of linear equations can have no solutions, one solution or infinitely many solutions
* If is in **row echelon** form, it will be easy to determine the solution(s)
* For any matrix, the first non-zero entry in a particular row is called the **leading entry** of that particular row
* A matrix is said to be in row echelon form if the leading entry in each row lies to the right of that in the preceding row

**Gaussian Elimination:** Method for reducing to row echelon form

* **ERO**s: (1) Multiply a row by a non-zero constant (2) Add or subtract a multiple of one row to or from another, respectively (3) Interchange two rows

**Linear Dependence and Independence**

* Consider a set of vectors and the equation ,
* Given the above, one of the following will always occur:
  1. , set of vectors is **linearly independent**
  2. Solution with at least one , set of vectors is **linearly dependent**

**Rank of a Matrix:** The **rank** of a matrix , denoted by , is simply the number of linearly independent rows in

**Recognising the Nature of Solutions**

* Consider a general system of linear equations in unknowns,
* i.e. is ­, and . Let be the augmented matrix for the system
  1. , equations are **inconsistent** and there are **no solutions**
  2. , equations are **consistent** and have a **unique solution**
  3. , equations are **consistent** and have **infinitely many solutions**, which we can express in terms of parameters

**Lecture 8: More on Linear Systems & Inverses**

|  |  |
| --- | --- |
| **Homogenous System of** **Equations in** **Variables** | **Matrix of System** |
| … |  |

* Note: is always a solution (the **trivial solution**), i.e. a homogenous system is always consistent
  1. , **trivial solution is the only solution**
  2. , **infinitely many solutions**. Includes **trivial solution** and **infinite non-trivial solutions**

**Calculating Inverses**

* To find the inverse of , form the augmented matrix , then apply EROs to this augmented matrix until the LHS turns into the identity. The resulting RHS will represent

**Invertibility and Solutions of Systems**

* Consider a system of equations in unknowns, i.e. , where is and
* Clearly, if is invertible with inverse , we have